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Matrix theory and N=(2,1) Strings

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We reinterpret N=(2,1) strings as describing the continuum limit of matrix theory with *all* spatial dimensions compactified. Thus they may characterize the full set of degrees of freedom needed to formulate the theory.

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Matrix theory [1] has been remarkably successful in capturing the essential ingredients of M-theory – U-duality groups for compactification on T^d for $d \leq 4$ (for a summary, see [2]), the asymptotic Coulomb potential of BPS-saturated sources, implementation of the 'holographic principle' [3,1], regularization of short-distance supergravity physics [4], and recovery of string theory when the compactification torus degenerates appropriately [1,5,6,7].

Nevertheless, the formulation is incomplete. At each stage of compactification, new ingredients are required. In particular, new degrees of freedom arising from additional wrapping modes of fluxes make their appearance. Due to the explosive growth of such modes, one reaches an impasse in defining matrix theory on T^6, and perhaps even on T^5 [2,8].

A seemingly different route to M-theory was initiated by D. Kutasov and the author [9-11], who showed that N=(2,1) heterotic strings reproduce the basic classes of M-theory branes in their target space dynamics. So far, this approach has received scant attention, perhaps due to the daunting prospect of using string 'field theory' to describe string worldsheets (although to be fair, the matrix theory formulation of string worldsheets involves an equivalent level of complexity – an infinite tower of massive modes also decouples from matrix dynamics in the limit that describes string theory). In addition, it has been difficult to see how the target string's interactions might be included in the (2,1) string approach.

Here, it is suggested that matrix theory and N=(2,1) strings cure one another's ills – matrix theory provides a natural framework for interpreting the results of [9-11] as well as for including interactions, while N=(2,1) strings provide the degrees of freedom needed to define compactification of all spatial dimensions in matrix theory. After briefly reviewing the salient facts about N=(2,1) strings, I describe the proposed correspondence for toroidal type II vacua, followed by a parallel treatment of heterotic/type I vacua. Then the role of U-duality is explored, in connection with a generalized Kac-Moody algebra (GKM) related to E_{10}.

N=(2,1) strings in brief

N=(2,1) heterotic strings couple a modified NSR superstring in the left-moving chiral sector with the self-dual, integrable structure of N=2 strings in the right-moving chiral sector. In a free field representation with

$$
\begin{aligned}
X_r^\mu \quad &, \quad \psi_r^\mu \qquad \mu = 0,1,2,3 \\
X_\ell^M \quad &, \quad \psi_\ell^M \qquad M = 0,1,\dots,11 \ ,
\end{aligned}
\tag{1}
$$

the gauge algebra is N=2 local supersymmetry on the right, with gauge currents T_r, $G_r^{\pm}$, J_r; and a reducible gauge algebra on the left, generated by N=1 local supersymmetry currents T_ℓ, G_ℓ together with the null supercurrent $J_\ell = v_M \partial X^M$, $\Psi_\ell = v_M \psi^M$. The right-moving coordinates have signature $(--++)$, the left-movers $(--++\ldots+)$. The current J_r specifies a selfdual two-form I via $J_r = \psi_r^\mu I_{\mu\nu} \psi_r^\nu$. The right-moving gauge constraints are sufficient to eliminate all nonzero modes of that chirality, leaving only the center-of-mass momentum p_r. We take the time directions noncompact, so that $p_\ell^0 = p_r^0$, $p_\ell^1 = p_r^1$; this ensures that the null constraints imposed by J_ℓ bring us to conventional Lorentzian signature dynamics. The spatial coordinates will be taken to be compactified on a torus, so that the spatial momenta lie on a Narain lattice $\Gamma^{10,2}$. It will be sufficient for our purpose here to specialize to $\Gamma^{10,2} = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right) \oplus \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right) \oplus \Gamma_8$.

The massless level vertex operators

$$
\begin{aligned}
V_{NS} &= (e^{-\phi_\ell - \phi_r^+ - \phi_r^-})\, \xi_M(p) \psi_\ell^M \; e^{ip_\ell \cdot X_\ell + ip_r \cdot X_r} \\
V_R &= (e^{-\frac{1}{2}\phi_\ell - \phi_r^+ - \phi_r^-})\, u_A(p) S_\ell^A e^{ip_\ell \cdot X_\ell + ip_r \cdot X_r}
\end{aligned}
\tag{2}
$$

satisfy the constraints $v \cdot \xi = p_\ell \cdot \xi = 0$, $\xi \sim \xi + \alpha p_\ell + \beta v$; $\not{v} u = \not{p}_\ell u = 0$; and $p_\ell^2 = p_r^2 = 0$. The only nontrivial S-matrix elements are the three-point functions

$$
\begin{aligned}
\langle V_{NS}(1) V_{NS}(2) V_{NS}(3) \rangle &= (\xi_1 \cdot \xi_2 \; p_2 \cdot \xi_3 + \text{cyclic})_\ell \times (p_1 \cdot I \cdot p_3)_r \\
\langle V_R(1) V_{NS}(2) V_R(3) \rangle &= (\bar{u}_1 \not{\xi}_2 u_3)_\ell \times (p_1 \cdot I \cdot p_3)_r \; .
\end{aligned}
\tag{3}
$$

The left-moving structure is that of the 9+1d vector supermultiplet. If the left- and right-moving momenta were completely independent, we would interpret the factor $(p_1 \cdot I \cdot p_3)_r$ as the structure constants of the gauge group. In fact the group would be that of symplectic diffeomorphisms of self-dual null planes in the 2+2 dimensions spanned by X_r^μ, which is well-known to be the symmetry group of self-dual gravity [12-14]. Formally, this $SDiff_2$ group can be thought of as SU(∞). Unfortunately, the left- and right-movers are coupled through the Virasoro constraints $p_\ell^2 = p_r^2 = 0$, due to the common zero-modes of the time components; this obstructs the naive interpretation of the massless sector of the (2,1) string as SU(∞) super Yang-Mills (SYM). Nevertheless, we will keep it in mind as a useful heuristic. It is likely that this $SDiff_2$ symmetry is present off-shell [15,16]. Finally, note that (2) are only the *massless* states; the full spectrum of physical states consists of the 'Dabholkar-Harvey' states [17] – right-moving ground states with arbitrary transverse left-moving excitation, subject only to the mild constraints of level-matching. The level density is exponential [17,11].

The correspondence with matrix theory

In [9], it was shown that there are generically two nontrivial decompactification limits of the (2,1) string, depending on the orientation of the null vector v_M. The simplest choice places v entirely within the 2+2 dimensions common to both left- and right-movers. In this case, the low-energy theory in the decompactification limit has the spectrum [9] and dynamics [11] of the type IIB D-string in static gauge. Let us compare this with the corresponding limit of matrix theory. The matrix theoretic description of M-theory on T^d is given by maximally supersymmetric Yang-Mills (SYM) on the dual torus $\tilde{T}^d$ [1,18]. Perturbative type IIA string theory on T^{d-1} arises from M-theory upon shrinking a circle to zero size. In the matrix SYM, the dual circle decompactifies; the low-energy dynamics is 1+1d SYM [1,5,6,7] – the spectrum and dynamics of the type IIB D-string!

The second nontrivial decompactification limit of the N=(2,1) string occurs when the null vector has its spatial component along one of the purely left-moving coordinates $X_4, ..., X_{11}$. Then the low-energy theory in the limit $R_2, R_3 \to \infty$ has the spectrum of the type IIA D2-brane [9]. The Lagrangian agrees to cubic order in the interactions [10]; to higher order the situation is not clear, although general arguments based on symmetries [11,19] (see also [20]) suggest that the target space dynamics must be physically equivalent to the D2-brane. In matrix theory, the decompactification of a $\tilde{T}^2$ in the SYM on $\tilde{T}^d$ yields the type IIB string [21,6,2]; the low-energy dynamics of the SYM is that of the type IIA D2-brane – again there is a direct parallel to the (2,1) string. Note also that the process stops here; the limit of a large $\tilde{T}^3$ is related to that of a small $\tilde{T}^3$ by U-duality (essentially by T-duality for membranes [22]). Small $\tilde{T}^3$ is *decompactification* in M-theory; one finds a nonperturbative theory on a higher dimensional spacetime (I will comment briefly on such limits below in a discussion of U-duality). Thus the two distinct weak-coupling IR limits of matrix theory parallel the two decompactifications of N=(2,1) string theory.

Matrix theory also provides a neat answer to the question of how to build interactions into the target dynamics of N=(2,1) strings. Matrix theory suggests [7] that they should be incorporated through operator insertions on a fixed background, rather than topology change of the background itself. It would of course be interesting to construct such operators for the (2,1) string.

Heterotic/type I vacua

The next basic class of string vacua are the toroidal compactifications of heterotic and type I strings. These are realized as M-theory on $T^d \times S^1/\mathbf{Z}_2$ [23]. There are several

ways to take the weak coupling, perturbative string limit. First, one may shrink a single circle to a size much smaller than the eleven-dimensional Planck scale. This yields either the $E_8 \times E_8$ heterotic or type IA string, depending on whether this 'M-theory circle' is the orbifold circle $S^1/\mathbf{Z}_2$ or one from T^d. Alternatively, one may shrink two circles to get a theory more closely related to IIB strings. Taking the orbifold circle as one of these, one obtains the type I/heterotic SO(32) dual pair (one or the other is weakly coupled depending on whether the radius $R_{S^1/\mathbf{Z}_2}$ is much smaller/larger than R_{S^1}). Taking both circles from T^d, one obtains type IIB with seven-branes – the F-theory description of the heterotic string on tori.

Some of the matrix-theoretic aspects of this family of vacua have been worked out [24]. Matrix quantum mechanics compactified on $S^1/\mathbf{Z}_2$ is described by 1+1 SYM on a dual circle $\tilde{S}^1$; the orbifold twist projects the SYM gauge group from U(N) to O(N), with the transverse coordinates X^i forming a matter multiplet in the symmetric tensor representation of O(N). The fermion spectrum surviving the projection (matrices θ^a in the adjoint and $\theta^{\dot{a}}$ in the symmetric tensor) is anomalous; this anomaly is cancelled by adding 32 fermion fields χ^I in the fundamental representation. Further toroidal compactification turns some of the X^i into covariant derivatives on the dual space. The fact that the original X^i are symmetric matrices means that the spectrum of the covariant derivatives is symmetric under reflection – the dual space is $\tilde{T}^d/\mathbf{Z}_2$. To summarize, M-theory on $T^d \times S^1/\mathbf{Z}_2$ is a matrix orbifold SYM on $\tilde{T}^d/\mathbf{Z}_2 \times \tilde{S}^1$.

Shrinking $S^1/\mathbf{Z}_2$ to get the heterotic string, the dual $\tilde{S}^1$ decompactifies. The IR description is that of a modified type I D-string [24]. The vacua described above, in which a two-torus contracts to zero size, correspond to IR limits of 2+1 SYM on a matrix orientifold $\tilde{S}^1/\mathbf{Z}_2 \times \tilde{S}$.

Precisely these sorts of SYM theories arise in the decompactification limits of a $\mathbf{Z}_2$ orbifold of the N=(2,1) string (see [10] for details). The orbifold twist reflects (X_r^1, X_r^3) and $(X_\ell^1, X_\ell^3, X_\ell^4, ..., X_\ell^{11})$, as well as the corresponding superpartners ψ.[1] The twist acts on the gauge algebra as

$$J_r \to -J_r \quad , \qquad G_r^+ \leftrightarrow G_r^-$$
$$J_\ell \to -J_\ell \quad , \qquad \Psi_\ell \to -\Psi_\ell \ .$$

(4)

One might view this twisted right-moving N=2 algebra as a reduction of the gauge group of symplectic diffeomorphisms from SU(∞) to SO(∞) at the fixed points. At the massless

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[1] In a convention where $J_r = \psi_r \cdot I \cdot \psi_r = \psi_r^0 \psi_r^3 + \psi_r^1 \psi_r^2$.

level, the twisted sector contains 32 massless chiral fermions, which for example generate the internal sector of heterotic strings.

Note that the twist acts on the left-moving 'spacetime' part of the string as though the space coordinates were on $\tilde{T}^9/\mathbf{Z}_2 \times \tilde{S}^1$; one of the $\mathbf{Z}_2$-twisted coordinates is removed by the null projection, leaving $\tilde{T}^8/\mathbf{Z}_2 \times \tilde{S}^1$. This would be appropriate to matrix theory with all transverse dimensions compactified. Decompactification yields the type I D-string on $\tilde{S}^1$, if the null vector points in the directions common to X_ℓ and X_r; and an orientifold D2-brane if the spatial component of the null vector points in the left-moving internal Γ_8. These are precisely the ingredients needed to describe perturbative string limits in the matrix formalism.

U-duality

Having motivated the link between N=(2,1) strings and matrix theory, it is appropriate to ask: Where is the U-duality group? A major success of matrix theory is that it accounts for the entire group of duality symmetries in high dimensions. Naively, one might begin by looking to classify fluxes of the SYM-like target space theory of (2,1) strings; identifying them with various Kaluza-Klein modes, wrapped branes, etc.; and resolving them into duality multiplets. However, this procedure will not work so simply in M-theory compactified to such a low dimension as seems apparent here. Consider the low-energy theory, namely 11d supergravity on T^9 (or similarly T^{10}). This theory has been analyzed in [25-29]; its bosonic sector consists of a scalar field manifold which is a loop group extension of $E_{8(8)}/SO(16)$, coupled to 2d gravity. Scalar fields are disordered in two dimensions due to strong IR fluctuations. There are no order parameters to orient the vacuum and provide a fixed background with respect to which one can consider the energies of various fluxes. Similar considerations apply to the T^{10} compactification, suspected to be related to E_{10} [25].

Instead, the theory consists of states, which are wavefunctions on the scalar coset manifold. The U-duality group enters by restricting the wavefunctions to be modular covariant functions under its action[2] (the analogue of restriction from functions on the upper half-plane $SL(2, \mathbb{R})/SO(2)$ to modular functions under $SL(2, \mathbf{Z})$). The wavefunctions might be interchanged by various U-duality transformations, forming a nontrivial vector bundle over the moduli space.

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[2] Thanks to J. Harvey for remarks clarifying this point.

Now, in discussions with G. Moore (as reported in [11]), it was realized that the partition function of the toroidally compactified N=(2,0) string (N=2 string on the right, bosonic string on the left) has as one-loop partition function the denominator formula of a generalized Kac-Moody algebra (in that case the Fake Monster Lie algebra, the natural GKM in 26 dimensions).[3] A property of this denominator formula is covariance under $O(26, 2; \mathbf{Z})$, the Narain group of the (2,0) string. The Narain moduli define a character on the root lattice of the GKM. We further conjectured that the N=(2,1) string realizes the natural generalization of E_{10} relevant to the superstring. Again one obtains modular properties under $O(10, 2; \mathbf{Z})$ and a similar interpretation of the Narain moduli. If this turns out to be the case, it would further cement the connection between maximally compactified M-theory and N=(2,1) strings, since this is the expected symmetry group. At the moment, our understanding of the quantum N=(2,1) string theory is too primitive to see its properties under the discrete U-duality group.

The replacement of a moduli space of distinct vacua by modular functions on moduli space has interesting cosmological implications [31,28]. These modular functions are essentially 'wave functions of the universe'. Their amplitudes in the cusps of the fundamental domain represent the probabilities of decompactification to various higher dimensions. Given the relation between fields and couplings in string theory, one also finds a natural probability measure on the space of coupling constants in these higher-dimensional theories. One would like to see if four dimensions is somehow preferred.[4] Higher-dimensional matrix theory compactifications would have to be recovered in these singular components. It would be very interesting to understand the implications for matrix theory on T^d for $d \geq 3$. It is by now well-established that a field-theoretic formulation of matrix theory must break down by the time one reaches five compact dimensions [2,8]. Beyond this it is not known how to proceed. N=(2,1) strings meet the requirements – a symplectic diffeomorphism gauge group structure, and SYM dynamics at low momenta; with a stringy spectrum of states to regulate the dynamics.

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[3] For a discussion of GKM algebras the context of string compactifications, see [30]

[4] In this vein, it was noted in [11] that the (2,1) string might serve as a prototype for an initial condition for cosmology.

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